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Probing semiclassical magneto-oscillations in the low-field quantum Hall effect

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The low-field quantum Hall effect is investigated on a two-dimensional electron system in an AlGaAs/GaAs heterostructure. Magneto-oscillations following the semiclassical Shubnikov-de Haas formula are observed even when the emergence of the mobility gap shows the importance of quantum localization effects. Moreover, the Lifshitz-Kosevich formula can survive as the oscillating amplitude becomes large enough for the deviation to the Dingle factor. The crossover from the semiclassical transport to the description of quantum diffusion is discussed. From our study, the difference between the mobility and cyclotron gaps indicates that some electron states away from the Landau-band tails can be responsible for the semiclassical behaviors under low-field Landau quantization.

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I. INTRODUCTION

Recently, there have been renewed interests toward the study of Landau quantization in two-dimensional electron systems (2DESs) under a perpendicular magnetic field B. It is well known that Landau quantization can modulate the density of states and induce magneto-oscillations periodic in the inverse of B in the longitudinal resistivity ρ_{xx}. At low enough B where the spin splitting is unresolved, such oscillations are expected to follow the Shubnikov-de Haas (SdH) formula such that[1,2]

$$\rho_{xx}(B,T) = \rho_{xx}(B=0) + \Delta\rho_{xx}\cos[\pi(\nu-1)] \tag{1}$$

with the oscillating amplitude

$$\Delta\rho_{xx}(B,T) \sim (X/\sinh X)F(B). \tag{2}$$

Here $\rho_{xx}(B=0)$ is the value of ρ_{xx} at $B=0$, ν represents the filling factor, and the parameter $X=2\pi^2 k_B m^* T/\hbar eB$ with k_B, e, $\hbar$, m^*, and T as the Boltzmann constant, electron charge, reduced Plank constant, electron effective mass, and temperature, respectively. The T-dependent factor $X/\sinh X$ comes from the Lifshitz-Kosevich (LK) formula[3,4] and the T-independent factor $F(B)$ is usually taken in the form embodied by the Dingle plot, i.e., the standard Dingle factor[1,5]

$$F(B) = 4c\rho_{xx}(B=0)\exp(-\pi/\mu_q B). \tag{3}$$

Here μ_q denotes the quantum mobility[6] and c is a numerical coefficient in the order of unity.[7–9] The SdH theory has been widely used to determine the effective mass, carrier concentration, and quantum mobility in semiconductor heterostructures.[10] Such a theory, including the LK formula, can be derived semiclassically without considering the quantum localization.[1] On the other hand, the quantum localization is important to the integer quantum Hall effect (IQHE).[11–13] On this account, electrons away from the broadened Landau-level center are considered to be localized and to be irrelevant to electrical conduction. Because of the localization effects, only states at centers of Landau bands are extended in the high-field IQHE.[13] It is believed that such states are distributed within a very narrow energy range.[14] The extended states in different Landau bands are separated by the mobility gaps ΔE. As the Fermi level is situated in the middle between adjacent Landau levels, ΔE can be evaluated at the minimum points of the longitudinal resistivity $\rho_{xx,\min}$ by[4,15]

$$\rho_{xx,\min} \simeq \rho_0 \exp(-\Delta E/2k_B T), \tag{4}$$

where the prefactor ρ_0 is independent of T. In the standard localization picture of the IQHE, the localized and the extended states are separated by a sharp boundary, called mobility edge. On the other hand, a network model for the magnetotransport of 2DES is recently proposed, from which a smooth change at the boundaries between localized and delocalized states is suggested.[16] Second-order phase transitions occur as Fermi energy passes through the extended states with sweeping B and universalities of such transitions have been investigated by studying the IQHE.[14,17,18]

Despite the success of the SdH and IQHE theories, more studies are still necessary to understand their applicable ranges. The quantum localization is taken into account in the standard IQHE theory even as $B\to 0$ but the localization length usually becomes much larger than the realistic sample size with decreasing B.[12,19,20] Therefore, the localization effects diminish at low fields and alternative mechanisms have been discussed to explain how the IQHE-like properties survive as such effects are reduced.[1,8,20–23] The semiclassical SdH theory, in fact, ignores the localization and works very well in explaining the low-field magneto-oscillations.[1] It is expected that the influences of the quantum localization gradually build up with increasing B, leading to the breakdown of Eq. (1) before[7,8,11]

$$\Delta\rho_{xx}(B,T) \geq \rho_{xx}(B=0) \tag{5}$$

and/or local minimum points of $\rho_{xx}\to 0$ with decreasing T. However, recent experimental studies showed that both Eqs. (2) and (3) can hold true even as $\rho_{xx}\to 0$,[7,8] and the interests in the applicable range of the semiclassical approaches are revitalized.[5,24] To understand the transport properties at intermediate magnetic fields, therefore, it is important to investigate the properties of both the semiclassical transport and quantum localization leading to the mobility gap.[1,11,20,21,25]

To further understand the low-field IQHE, in this paper we present a magnetotransport study on the 2DES in an AlGaAs/GaAs heterostructure. In addition to the extension of Eqs. (2) and (3),[7,8] we found semiclassical behaviors can coexist with the activation law, which indicates the quantum localization. The value of ΔE, in fact, supports that the mobility gap is due to the electronic states in the Landau-band tails[16] while the semiclassical behaviors may be attributed to some states away from the tails. Therefore, we shall consider different types of electronic states to understand the survival of semiclassical formula under the quantum localization which leads to the mobility gap. With further increase in magnetic field, we found that the semiclassical LK formula remains valid even as the oscillating amplitude is so large that corrections to the Dingle factor could be attributed to the quantum diffusion.[21]

In the following report, the observations of magneto-oscillations and the analysis based on Eqs. (2) and (3) are presented in Sec. II A. We demonstrate in Sec. II B the existence of a mobility gap by thermally activated conductivity measurements. The mobility gap, which is shown to be related to localization under Landau quantization, can coexist with semiclassical behaviors. Based on our experimental outcome, a possible picture for the crossover from semiclassical transport to the quantum Hall effect is presented in Sec. III. In Sec. IV, we discuss the pre-exponential factor in the thermal activation law as well as the crossover from the Dingle factor to the quantum diffusion model. Conclusions are given in Sec. V.

II. EXPERIMENT AND ANALYSIS

A. Magneto-oscillations at low magnetic fields

The sample used for our study is a modulation-doped AlGaAs/GaAs heterostructure grown by molecular-beam epitaxy. The 2DES under study resides in the GaAs side of the heterojunction. The two-dimensional (2D) channel was followed by a 20 nm spacer layer of $\mathrm{Al}_{0.28}\mathrm{Ga}_{0.72}\mathrm{As}$, a 40 nm layer of graded $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ (with x from 0.28 to 0) doped with Si at 1×10^{18} cm^{-3}, and a 12 nm GaAs cap layer doped at 1×10^{18} cm^{-3}. The sample was made into a Hall pattern of 0.4 mm width with voltage probes spaced 1 mm apart. Magnetotransport measurements were done with a 14 T superconducting magnet and a He^4 refrigerator. Magnetoresistance under reversed current is measured to eliminate the thermal voltage.

The curves of ρ_{xx} at different temperatures in the field region $B=0-2.6$ T are shown in Fig. 1. At low magnetic fields, the 2DES behaves classically so that ρ_{xx} remains constant. Magneto-oscillations can be observed in ρ_{xx} as we gradually increase the perpendicular magnetic field B. From the oscillating period with respect to $1/B$, the carrier concentration $n=4.7\times10^{11}$ cm^{-2} can be obtained. The scattering mobility μ_c obtained by $\rho_{xx}(B=0)=1/ne\mu_c$ is 5.6×10^5 cm^2/Vs.

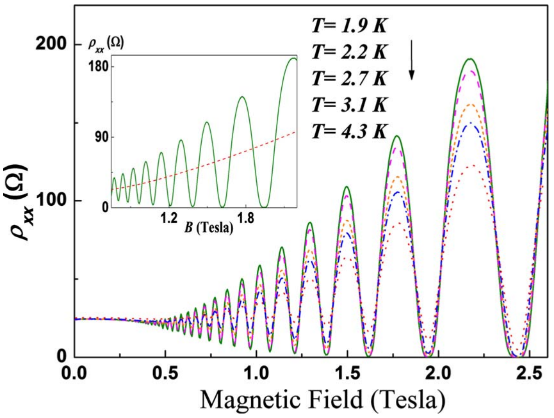

FIG. 1. (Color online) The longitudinal resistivity as a function of magnetic field for different temperatures (from top to bottom are 1.9, 2.2, 2.7, 3.1, and 4.3 K). The thermal offset has been removed by employing reverse current measurements. The dashed line in the inset shows the nonoscillatory background at $T=1.9$ K obtained by averaging the magneto-oscillations.

The carrier effective mass is a quantity that can be quantitatively deduced from the semiclassical SdH theory. It is well established that the effective mass m^* in 2D GaAs electron gases is $0.067m_0$. Thus, in order to probe the validity range of the SdH theory, we can investigate Eqs. (1)–(3) under the expected effective mass. We note that as X is large enough such that $X/\sinh X \sim 2X\exp(-X)$ in Eq. (2), the reduced form can be expressed by

$$\ln[\Delta\rho_{xx}(B,T)/T] \sim C - 2\pi^2 k_B m^* T/\hbar eB, \tag{6}$$

where $C=\ln[2\pi^2 k_B m^* F(B)/\hbar eB]$ is a parameter independent of T. Therefore, we can first probe Eq. (2) by checking whether the slope of $\ln(\Delta\rho_{xx}/T)-T$ yields the effective mass m^* close to the expected value. In our study, the effective mass is about $0.067m_0$ from the slope as $B<1.3$ T, where X is large enough to validate Eq. (6). For example, as shown in the inset to Fig. 2, we have $m^*\sim0.069m_0$ from the slope of $\ln(\Delta\rho_{xx}/T)-T$ at $B=0.805$ T. To exactly examine Eq. (2) at a specific B, we can check $\Delta\rho_{xx}=F(B)X/\sinh X \propto X/\sinh X$, the expected LK factor, with $m^*=0.067m_0$. The dashed line in Fig. 2 shows the fitting at $B=0.805$ T. The good fitting in our study justified the extraction of the T-independent factor $F(B)$. To further examine the LK factor for all resolved magneto-oscillations, we comprehensively compare $\Delta\rho_{xx}/F(B)$ and $X/\sinh X$ in Fig. 3. As can be seen in Fig. 3, the ratio of $\Delta\rho_{xx}/F(B)$ collapses to $X/\sinh X$ very well, indicating that the LK formula holds true in our study. Even as $B>1$ T, the consistency still retains although the expected criterion for breakdown, i.e., Eq. (5) becomes valid. In addition, the collapse in Fig. 3 indicates the validity of the LK formula when Eq. (6) fails as $X<1$. Therefore, the semiclassical LK formula is applicable even when Eq. (5) holds true, which is consistent with previous reports.[7,8]

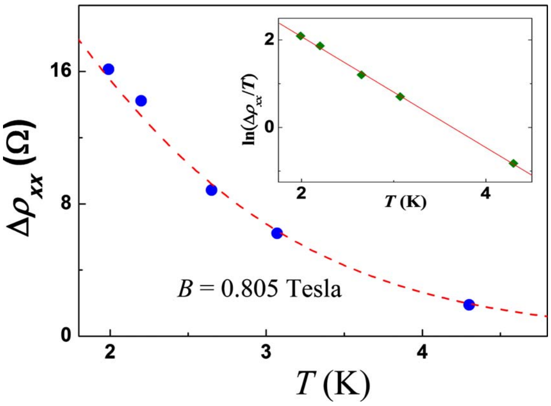

FIG. 2. (Color online) The curve of $F(B)(X/\sinh X)$ is represented by the dashed line and the experimental points of $\Delta\rho_{xx}$ at $B=0.805$ T are indicated by the solid circles. It is found that Eq. (2) is valid at this field strength. The inset displays the plot $\ln(\Delta\rho_{xx}/T)$ versus T at $B=0.805$ T. According to the LK factor, the carrier effective mass can be extracted from the slope, which is close to the expected value.

The factor $F(B)$ is usually taken as the standard Dingle factor at low magnetic fields. Equation (3) can be rearranged as

$$\ln[F(B)/\rho_{xx}(B=0)] = \ln(4c) - \pi/\mu_q B. \tag{7}$$

As shown in Fig. 4, $\ln[F(B)/\rho_{xx}(B=0)]$ is linear with respect to $1/B$ as $B<1.25$ T. The quantum mobility μ_q can be extracted from the slope of $\ln[F(B)/\rho_{xx}(B=0)]$ with respect to $1/B$, yielding $\mu_q=3.5\times10^4$ cm^2/Vs. The constant $c=1.2$ is deduced from the intercept. As $B>1.25$ T, we can see the deviation to Eq. (7) while the LK formula is still applicable. Our study reveals that the LK formula can hold true alone even when we shall consider the corrections to SdH theory to refine Eq. (3). This is going to be discussed in Sec. III.

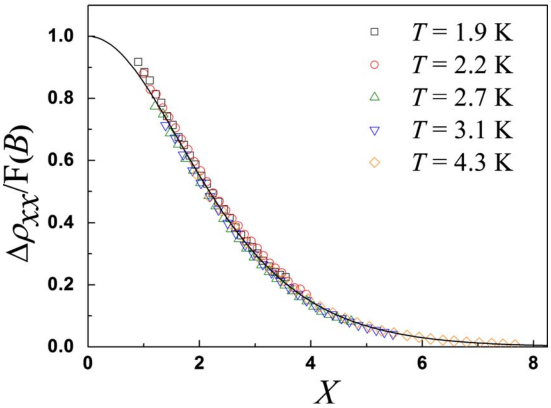

FIG. 3. (Color online) To check the LK formula for all resolved magneto-oscillations, we comprehensively plot $\Delta\rho_{xx}/F(B)$ with respect to X for various fixed temperatures. The symbols squares, circles, up triangles, down triangles, and diamonds are for the points at $T=1.9$, 2.2, 2.7, 3.1, and 4.3 K, respectively. The range of magnetic field extends to 2.17 T. The numerical evaluation of $X/\sinh X$ as a function of X is shown as the solid line. The curve of $\Delta\rho_{xx}/F(B)$ collapses well into a single curve $X/\sinh X$ with respect to the parameter X.

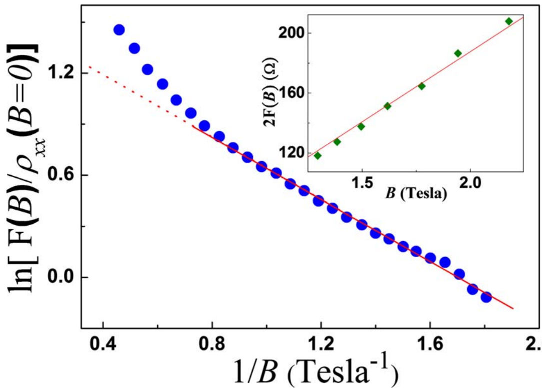

FIG. 4. (Color online) $\ln[F(B)/\rho_{xx}(B=0)]$ as a function of inverse magnetic field from which the quantum mobility can be obtained. The inset shows $2F(B)$ as a function of magnetic field and the fitting to Eq. (11).

B. Mobility gap

The formation of the mobility gap is expected under the quantum localization, which is important to the IQHE under the high-field Landau quantization. Figure 5(a) shows the expected density of states in the spin-degenerate IQHE under a perpendicular magnetic field B, where the cyclotron gap $\hbar eB/m^*$ separates the centers of adjacent Landau bands. At high B, all the electrons are localized except those in the white region of the width Γ_e near the center of each band. The localized electrons are irrelevant to the conductivity, so an excitation for the change in ρ_{xx} must overcome a mobility gap ΔE to modify the distribution of the conducting electrons. At low temperatures and/or large effective size, the equation

$$\Delta E \sim \hbar eB/m^* \tag{8}$$

is expected because Γ_e is very small. At suitable temperature range where Eq. (4) is valid, we have

$$\ln \rho_{xx,\min} = \ln \rho_0 - \Delta E/2k_BT \tag{9}$$

at the minimum points of ρ_{xx} in B. When the spin splitting is unresolved, such points correspond to the even filling factors

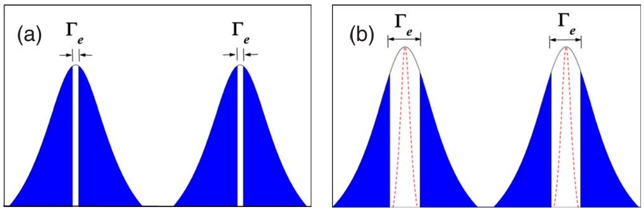

FIG. 5. (Color online) (a) Density of states in Landau bands for spin-degenerate 2DES in a perpendicular magnetic field. At high B, only electrons in the white regions of width Γ_e near the center of each band are delocalized. (b) For the intermediate field strength, only electrons in the tails, shown as black regions, are well localized. The mobility gap is consequently reduced. The dashed line shows the schematic distribution of electrons responsible for the semiclassical transport.

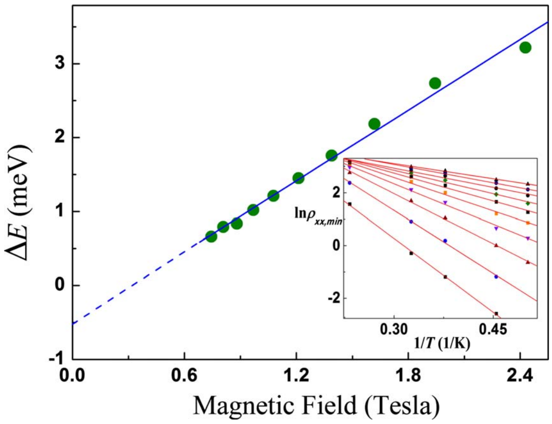

FIG. 6. (Color online) The activation energies as a function of magnetic field. The solid line is the linear fit to the data. The zero-field intercept is indicated by the dashed line. The inset shows the Arrhenius plot of $\rho_{xx,\min}$ with linear fits (from top B=0.745, 0.808, 0.881, 0.969, 1.077, 1.212, 1.389, 1.619, 1.944, and 2.429 T).

and the Fermi energy E_F is expected to be located near the middle point between adjacent Landau bands in Fig. 5(a).

The mobility gap at $B=0.74-2.43$ T is obtained from the fitting according to Eq. (9) at the even filling factors from 8 to 26 under suitable temperature range, as shown in the inset to Fig. 6. The good linear fitting of ln $\rho_{xx,\min}$ with respect to T^{-1} indicates the existence of the mobility gap in such a magnetic-field region, where the semiclassical LK formula or Eq. (2) holds true as shown in Fig. 3. Figure 6 shows the obtained ΔE as a function of the magnetic field. We can see that the gap ΔE is linear in B with the slope 1.6 meV/T, which is 92% of $\hbar e/m^*$ as estimated by Eq. (8) with $m^*=0.067m_0$, the well-established value in the 2D GaAs electron gases. Hence the slope of ΔE-B provides the quantitative evidence for the mobility gap to be attributed to Landau quantization. In our study, such a gap exists while the LK formula applies. Besides, between $B=0.74\sim1.25$ T, the standard Dingle factor given by Eq. (3) can also be fitted well, which is accompanied by the formation of the gap. Therefore, we found the coexistence of the mobility gap, the semiclassical LK formula, and the Dingle factor in the low-field IQHE before the ultimate deviation to semiclassical descriptions in strong magnetic fields.

In our study, we investigate the LK formula as $X>1$. Without numerical factors this condition translates into $k_BT>\hbar eB/m^*$ with $\hbar eB/m^*$ as the cyclotron gap. On the other hand, for the activation law, Eqs. (8) and (9), to hold true, one needs the opposite condition, $k_BT<\hbar eB/m^*$. From the first glance, the regime for the above-mentioned coexistence seems to be absent, which is opposed to the experimental outcome. Here, the solution to the dilemma is, in fact, provided by a large factor $2\pi^2$ in the definition of X. It is this factor that makes X large enough for our analysis of the LK formula. Therefore there exists parametrical window for both equations to be valid in our investigation.

III. FROM SEMICLASSICAL TRANSPORT TO QUANTUM HALL EFFECT

In the crossover from semiclassical transport to the IQHE, the semiclassical LK formula and Dingle factor appearing in the SdH transport theory can survive when the formation of the mobility gap indicates the importance of the quantum localization. To explain the unexpected coexistence in the crossover, we note that the semiclassical approach works best when Landau quantization modulates the density of states without inducing noticeable quantum localization at low enough B in most realistic 2DESs. It is essential that when the localization effects gradually take over with increasing B, the localization length is not the same for all states. Because the localization length increases rapidly near each Landau-band center,[14,20] the onset of the localization should emerge from the tails of each Landau band. In the intermediate magnetic-field region, as shown in Fig. 5(b),[16] it is possible that only tails of Landau bands, shown as black regions, are fully occupied by localized electrons. Meanwhile, the mobility gap is reduced because of the broadening of the width Γ_e due to the insufficient localization, shown as the white regions in which the electrons are not all localized. Moreover, the survival of the semiclassical LK formula under the apparent formation of the mobility gap could be attributed to a distribution of the semiclassical conducting electrons away from the Landau-level tails.[16] The nonzero intercept in Fig. 6, in fact, indicates $\Delta E\sim\hbar eB/m^*-\Gamma_e$ and provides the quantitative width value $\Gamma_e=0.51$ meV for the white regions in Fig. 5(b). In our study, the value of Γ_e is in good agreement with the usual broadening measure $\hbar/\tau_q=0.50$ meV, where the quantum lifetime $\tau_q=\mu_q m^*/e$.

With further increase in magnetic field, the oscillation amplitude becomes comparable to the zero-field resistivity value. The longitudinal resistivity goes to zero at its minima, which is regarded as a characteristic of IQHE. Here we start to see the deviation of the semiclassical approach in the Dingle plot as $B>1.25$ T. However, as Fig. 3 shows, the LK formula remains surprisingly valid with the existence of thermal activated conductivity due to quantum localization. Therefore we provide firm evidence that the LK formula is more robust than the Dingle factor with respect to quantum localization. As shown in the inset to Fig. 4, we found that $F(B)\propto B$ when there exists the deviation to the Dingle factor. We note that Coleridge[21] has derived the equation

$$\rho_{xx}^{peak}=\rho_{xx}(B=0)\mu_q B \tag{10}$$

for the deviation by considering the quantum diffusion effects. Here ρ_{xx}^{peak} denotes the peak value of ρ_{xx} at low temperatures. Because the minimum of ρ_{xx} approaches zero with decreasing T when Eq. (3) fails, we can expect

$$2F(B)\sim\rho_{xx}^{peak}=\rho_{xx}(B=0)\mu_q B\propto B \tag{11}$$

as $T\to0$ such that $X/\sinh X\to1$. Therefore, the experimental result $F(B)\propto B$ is consistent with Eq. (10). Besides, the slope of $2F(B)-B$ in our study is only 10% larger than the product $\rho_{xx}(B=0)\mu_q$, which provides the quantitative evidence to the validity of Eq. (10). In view of the deviation to the Dingle factor shown in Fig. 4, our results suggest the importance of the quantum diffusion to the corrections of the Dingle factor in the departure from the semiclassical transport.

IV. DISCUSSION

Conventional description for the quantum magneto-oscillations is based on the semiclassical model in which the discrete zero-field 2D density of states are evolved into broadened Landau levels. Note that the quantum localization plays no role in this approach. When the modulation to the density of states is not large at low magnetic fields, the oscillation amplitude is small and the analytic formulas given by Eqs. (1)–(3) can be derived from the semiclassical SdH theory. As the oscillation amplitude gradually increases with increasing B, this description is expected to be invalid. However, our analysis shows that both the LK factor and the Dingle factor hold true to a field strength larger than expected. Hence the applicable range of the semiclassical description can be extended beyond the region implied by the conventional derivation.[7,8,24] From a practical point of view, this suggests that the condition for magneto-oscillation analysis may be less stringent than general belief. Because of the positivity of ρ_{xx}, Eq. (1) cannot hold true when Eq. (5) is valid. It has been proposed that Eq. (1) should be refined by incorporating the positive magnetoresistance background for the extension of Eqs. (2) and (3).[7,8] Consistent with this point of view, the dashed line in the inset to Fig. 1 shows such a background at $T=1.9$ K. In addition to the nonoscillatory background, there also exists distortion on the oscillating factor $\cos[\pi(\nu-1)]$ in Eq. (1) as the minimum of ρ_{xx} approaches zero in our study. Different mechanisms have been discussed to understand the deviations on the semiclassical SdH theory given by Eqs. (1)–(3) as the magnetic field is increased.[1,11,20,21,25]

In addition, we found the Dingle plot in Fig. 4 yields the constant $c=1.2\sim 1$, the expected value for the Dingle factor. The small deviation of the constant c to the expected value,[7–9] in fact, is important to the crossover from the Dingle factor to Eq. (11). A direct crossover can occur when

$$F(B) = 4c\rho_{xx}(B=0)\exp(-\pi/\mu_q B) = \frac{1}{2}\mu_q\rho_{xx}(B=0)B. \tag{12}$$

If $c=1$ exactly, the second equality fails and there should be no direct crossover. The direct crossover in our study is evidenced by the fact that both Eqs. (3) and (11) apply between $B=0.74-1.25$ T. As the magnetic field is additionally increased in the crossover, the factor $F(B)$ approaches Eq. (11) while it departs from Eq. (3). We note that the current carried by the edge channels,[26,27] the voltage drop near the current injection points,[28] and the shape of Landau bands[21] are all important to the details of magnetotransport. Hence more studies are necessary to further understand the criteria for the validities of Eqs. (2) and (3).

From a detailed theoretical perspective, Eqs. (1) and (2) are based on the lowest-harmonics formula. When the magnetic field increases to certain level, higher Shubnikov-de Haas harmonics may begin to play a part.[29] At this time we cannot exclude the possibility that higher Shubnikov-de Haas harmonics may be responsible for the deviation of $F(B)$ from the linear dependence in Fig. 4. Thus further theoretical investigations shall be of high value for the understanding of magneto-oscillations in the low-field quantum Hall effect.

It is known that the mobility gap ΔE in Eq. (4) can deviate from the cyclotron gap under the variations on the positions of extended states,[30] electron-electron interaction,[31] and the existence of quasiparticles.[25] For the validity of Eq. (8), the density of states in the tails of the Landau bands should be significant, i.e., the Landau bands are not well separated in our 2DES. This may be in contrast with the case in which the Landau-band width is much smaller than the cyclotron gap such that there is no allowed state in the middle region of the cyclotron gap. In this case, the chemical potential is a strong function of magnetic field. When the concentration of electrons is fixed, the chemical potential resides within the Landau band instead of in the middle of the cyclotron gap. The activation energy is given by the distance from the chemical potential to the nearest band of delocalized states and hence is much smaller than $\hbar eB/m^*$. The value of ΔE in our study is close to the expected cyclotron gap with increasing B, which indicates the significant tail states in Landau bands. The energy $\hbar/\tau_q=\hbar e/\mu_q m^*$ determined by the Dingle factor or by the quantum diffusion model is close to the zero-field intercept of the solid line in Fig. 6. Such an intercept, in fact, indicates that the mobility gap disappears as the cyclotron energy becomes close to the broadening measure $\hbar/\tau_q$ with decreasing B, as reported in Ref. 32.

Finally, we discuss the pre-exponential factor in the equation for thermal activation. By using the Hall resistivity $\rho_{xy}=h/\nu e^2$ for quantum Hall state of filling factor ν, Eq. (4) can be converted into the equation for longitudinal conductivity σ_{xx}. We then have

$$\sigma_{xx} \simeq \sigma_0 \exp(-\Delta E/2k_B T) \tag{13}$$

with the prefactor $\sigma_0=\rho_{0,\nu}/(h/\nu e^2)$. Different experimental results on σ_0 have been reported.[33–35] For GaAs/AlGaAs heterostructures, Clark *et al.*[33] reported σ_0 value close to e^2/h at $\nu=2$, 4, and 6 while σ_0 value $\sim 2e^2/h$ at $\nu=2$ has been obtained by Usher *et al.*[34] In our study, the pre-exponential factor is found to be $\sim 2e^2/h$ at $B=0.74$ T. As the magnetic field increases, we found σ_0 gradually drops to $\sim e^2/h$. A similar result has also been reported for a SiGe/Si quantum well in which σ_0 is found to be $1.2e^2/h \sim 2e^2/h$ between $\nu=4$ and 16.[35]

V. CONCLUSION

In conclusion, we have performed a magnetotransport measurement on a modulation-doped AlGaAs/GaAs heterostructure. The behaviors from semiclassical transport to quantum localization are investigated. The extended applicable range of the LK formula is carefully demonstrated. The activation energy study indicates that the origin of the mobility gap is attributed to the localization under Landau quantization. In addition, we found the coexistence of the LK formula and the Dingle factor with the mobility gap in the crossover. The survival of the semiclassical LK formula under the mobility gap can be attributed to a distribution of conducting electrons away from the Landau-band tails. While the LK formula remains valid with large oscillating amplitudes, we show that the departure from the Dingle factor implies a smooth transition to the quantum diffusion model. Our results suggest that different types of electronic states should be taken into account to understand the coexistence of the semiclassical transport and quantum localization.

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